Short-time critical dynamics of the two-dimensional random-bond Ising model

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With Monte Carlo simulations we investigate the nonequilibrium critical dynamic behavior of the two-dimensional random-bond Ising model. Based on the short-time dynamic scaling form, we estimate all the static and dynamic exponents from dynamic processes starting with both disordered and ordered states. Corrections to scaling are carefully considered.

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I. INTRODUCTION

Ferromagnetic systems with quenched randomness have been studied intensively in recent years. For such systems, a main subject is whether the quenched randomness changes the universal class of the phase transition. In 1974 Harris had proposed a criterion [1]: if the critical exponent α is positive for the pure system, the quenched randomness changes the critical exponents, but if α is negative, the universal class of the disordered system remains the same. This criterion works well for most systems. However, for the two-dimensional (2D) Ising model where $\alpha=0$, one cannot draw a definite conclusion. Theoretical analysis predicted that for the 2D Ising model the randomness could only induce a logarithmic correction to the critical behavior, and all the critical exponents are not changed [2–5]. For example, in the critical region the following behavior has been proposed for the correlation length:

$$\xi \sim \tau^{-\nu}[1+C\ln(1/\tau)]^{\tilde{\nu}}, \quad \nu=1, \quad \tilde{\nu}=\frac{1}{2}, \tag{1}$$

where $\tau=(T-T_c)/T_c$ is the reduced temperature.

Numerical studies of influence of quenched randomness on the 2D Ising model have been carried out by Monte Carlo simulations and by transfer matrix calculations [6–16]. Some of these numerical studies favor the theoretical prediction; i.e., the quenched randomness could only induce a logarithmic correction to the critical behavior [6–11]. However, some other Monte Carlo studies of these models support a weak universality scenario: while η and γ/ν remain the same as those of the pure system, the exponents ν and γ change with the strength of randomness [14–16]. The key point here is that in order to extract the critical exponents from the simulations in equilibrium, one has to apply the finite size scaling analysis, and different schemes of such an analysis could favor different fittings when the data are not sufficiently accurate.

In the last several years, nonequilibrium critical dynamics has been developed essentially. After carefully taking into account the effect of the *macroscopic* initial states, a dynamic scaling form can be written that is already valid in the macroscopic *short-time* regime [17–27]. The short-time dynamic scaling presents many new phenomena compared with the long-time dynamic scaling. An interesting example is the so-called critical initial *increase* of the magnetization [17,21–23]. It shows that the initial conditions can induce rather anomalous behavior. More importantly, the static critical exponents and the dynamic exponent z originally defined in equilibrium or the long-time regime of the dynamic evolution, appear in the short-time dynamic scaling form. This fact might not look highly nontrivial but practically it leads to new methods for the numerical measurements of all the static and dynamic critical exponents as well as the critical temperature; for a review, see Ref. [24]. Since the measurements now are carried out in the short-time regime of the dynamic evolution, they do not suffer from critical slowing down. Because of the small nonequilibrium spatial correlation length, it is also easy to overcome the finite size effect. Compared with those methods, e.g., the nonlocal cluster algorithms, developed in equilibrium to overcome critical slowing down, the dynamic approach does study the original *local* dynamics. Furthermore, it could apply to systems with quenched randomness where the nonlocal cluster algorithms usually meet difficulties.

To observe the dynamic scaling in the macroscopic short-time regime, we have to wait the time t_{mic} that is sufficiently long in the microscopic sense. t_{mic} is the time the system needs to sweep away the microscopic details. Of course, t_{mic} is not universal and essentially depends on the microscopic details. In Monte Carlo simulations with local algorithms, for example, if a sweep over all the lattice sites is considered to be a typical microscopic time unit, and the interaction is only nearest neighbor or next nearest neighbor, t_{mic} should be about 10–100 time steps. In most of the numerical simulations, this is indeed the case [24]. However, there are examples that even after 100 time steps, the scaling behavior is still not so clean. In less severe cases, the deviation of the exponents is only about 1% or 2% from the expected values [28,23,29]. Taking a somewhat larger t_{mic} or an inverse power law correction, one can improve the results efficiently. In more severe cases, the effective exponents can be 10–20 % different. In the last case, the difficulty will not be easily removed by simply measuring in slightly later times. For the 2D XY model, for example, it is not clear whether the dynamic scaling is initial state dependent or not [30]. Recently, it has been argued analytically that a logarithmic correction should be responsible for this deviation [31], even

though the presented numerical data could not uniquely support it. It is believed that the vortex pairs are responsible for the logarithmic correction.

Large corrections to scaling in the short-time critical dynamics also appear in statistical systems with quenched randomness—our data in this paper show this. The origin should be the many metastable states. When the system starts from a disordered initial state, the relaxation to the equilibrium state at the critical temperature or nearby is affected severely by these metastable states. This situation seems to be similar to that in the 2D XY model where vortex pairs play an essential role. In the real world, systems with quenched randomness are very important. Therefore, the purpose of this paper is to study systematically the short-time critical dynamic behavior of systems with quenched randomness taking the 2D random-bond Ising model as an example. Special attention is drawn to the possible corrections to scaling. After excluding both dynamic and static corrections to scaling, accurate critical exponents will be obtained. It is interesting to see whether and how both static and dynamic exponents change along the critical line. Such a thorough and systematic study is crucial for further application of the short-time dynamics to disorder systems.

In the following section, we introduce the model and short-time dynamic scaling. In Secs. III and IV, results of the simulations from both the ordered and disordered initial states are presented, respectively. Finally the conclusion follows.

II. SHORT-TIME DYNAMIC SCALING

A. The model

In this paper, we report our systematic results of Monte Carlo simulations for the short-time critical dynamic behavior of the 2D random-bond Ising model on a square lattice. The Metropolis algorithm is used in simulations. A time unit is defined as a sweep over all spins on a lattice. The Hamiltonian of the model is

$$-H/kT=\sum_{\langle ij\rangle} K_{ij}S_iS_j, \tag{2}$$

where S_i is the Ising spin and the sum is over the nearest neighbors. The couplings K_{ij} are taken as K or rK randomly with probability 1/2. The critical point K_c is given by the self-dual relation [32]

$$\sinh(2K_c)\sinh(2rK_c)=1. \tag{3}$$

Simulations have been carried out for different strengths of randomness, $r=0.5$, $r=0.25$, and $r=0.1$ and for different initial conditions. In all cases, dynamic scaling behavior are examined.

B. Short-time dynamic scaling

For dynamic behavior of critical systems, traditionally it is believed that universal scaling exists only in the long-time regime of the time evolution. However, in recent years it has been discovered that starting from some *macroscopic* initial states, universal scaling behavior emerges already in the macroscopic short-time regime of the dynamic process [17–19,21,22,24], after a microscopic time scale t_{mic}. A typical example is a magnetic system initially in a high temperature state with a *small* initial magnetization m_0, suddenly quenched to the critical temperature T_c or nearby (without external magnetic field) and then released to a dynamic evolution of model A [33,34]. A generalized dynamic scaling form can be written down, for example, for the kth moment of the magnetization,

$$M^{(k)}(t,\tau,L,m_0)=b^{-k\beta/\nu}M^{(k)}(b^{-z}t,b^{1/\nu}\tau,b^{-1}L,b^{x_0}m_0). \tag{4}$$

Here t is the time variable, τ is the reduced temperature, L is the lattice size, β and ν are standard static exponents, and z is the dynamic exponent. Important is that a new independent exponent x_0 is introduced to describe the scaling behavior of the initial magnetization m_0. If the scaling form above is valid, in principle all relevant exponents can be extracted from the short-time behavior of suitable observables.

From Eq. (4), neglecting the finite size effect and noting that m_0 is small, it is easy to derive that at the the initial stage of the time evolution, the magnetization at the critical temperature presents a universal power law behavior,

$$M(t,m_0)\sim m_0t^{\theta}, \quad \theta=(x_0-\beta/\nu)/z. \tag{5}$$

Numerical results and analytical calculations have revealed that the exponent θ is positive for most systems, i.e., the magnetization undergoes *an initial increase*. The physical mechanism for this increase has not been clear. At least the mean-field effect or symmetry breaking is not very relevant.

Taking into account that the nonequilibrium spatial correlation length ($\sim t^{1/z}$) is small at the initial stage of the time evolution, the second moment of the magnetization at T_c subjects to a finite size scaling

$$M^{(2)}(t,L)\sim L^{-d}t^{y}, \quad y=(2-\eta)/z. \tag{6}$$

For simplicity, here we have assumed $m_0=0$.

Another interesting observable is the autocorrelation

$$A(t)\equiv\frac{1}{L^d}\left\langle\sum_i S_i(0)S_i(t)\right\rangle. \tag{7}$$

At the critical temperature T_c and $m_0=0$, $A(t)$ decays by a power law [35]

$$A(t)\sim t^{-\lambda}, \quad \lambda=\frac{d}{z}-\theta. \tag{8}$$

What is interesting here is that even though we have set $m_0=0$, the exponent θ (i.e., x_0) still enters the autocorrelation. This is because x_0 is actually the scaling dimension of the *local* magnetization. For details of the above scaling analysis and a more systematic extension, readers are referred to Refs. [17,35] and the review paper Ref. [24].

On the other hand, starting from an ordered initial state, dynamic scaling behavior of the system can be described by the scaling form

$$M^{(k)}(t,\tau,L)=b^{-k\beta/\nu}M^{(k)}(b^{-z}t,b^{1/\nu}\tau,b^{-1}L). \quad (9)$$

This scaling form looks the same as that in the long-time regime but it is now assumed to hold already in the macroscopic short-time regime. Many numerical simulations show that if we are only interested in determining the static exponents and the dynamic exponent z, this dynamic process is more favorable, since the statistical fluctuation is less severe.

At the critical temperature and for sufficiently large lattices the magnetization M decays by a power law

$$M(t)\sim t^{-\beta/\nu z}. \quad (10)$$

In order to obtain the critical exponent $1/\nu$, we assume that τ is slightly different from zero. Then the power law behavior of $M(t)$ is modified by a scaling function, i.e., $M(t,\tau)=t^{-\beta/\nu z}G(t^{1/\nu z}\tau)$. Differentiation of this equation leads to

$$\partial_\tau \ln M(t,\tau)\big|_{\tau=0}=t^{1/\nu z}\partial_{\tau'}\ln G(\tau')\big|_{\tau'=0}. \quad (11)$$

Finally, to determine the dynamic exponent z independently, we introduce a time-dependent Binder cumulant $U(t,L)=M^{(2)}(t,L)/M^2(t,L)-1$. From simple finite size scaling analysis based on Eq. (9), we can easily deduce the power law behavior of $U(t,L)$ at T_c,

$$U(t,L)\sim t^{d/z}. \quad (12)$$

Since the critical temperature for the 2D random-bond Ising model is exactly known, in this paper we will not discuss how to locate it. For details, readers are referred to Ref. [24].

III. RELAXATION FROM AN ORDERED INITIAL STATE

In this section, we present results of simulations starting from an ordered initial state. Statistical fluctuation in this process is usually less severe than that from a disordered initial state. Furthermore, up to now, a strong correction to scaling has not been observed, in contrast to those processes starting from disordered initial states in the cases of XY systems or disordered systems. Therefore, this dynamic process is favorable for the determination of the static exponents and dynamic exponent z.

We have carried out simulations with the Metropolis algorithm from an ordered initial state with the strength of the randomness $r=0.5$, 0.25, and 0.1, respectively. In order to obtain the derivative $\partial_\tau \ln M(t,\tau)|_{\tau=0}$, for each r we have performed simulations at three different temperatures in the critical region, $K=K_c$ and $K_\pm=K_c\pm\Delta K$. The derivative $\partial_\tau \ln M(t,\tau)|_{\tau=0}$ can then be approximated as $[M(t,K_+)-M(t,K_-)]/M(t,K_c)$ with an error of the order $O([\Delta K]^2)$. In the simulations we mainly set $\Delta K=0.005K_c$. To confirm that this ΔK is small enough, we have also carried out extra simulations with $\Delta K=0.0025K_c$ for $r=0.25$. The lattice size is $L=128$, and the system is updated up to 1000 Monte Carlo steps. An average is taken over 300 realizations of the couplings, and for each realization 100 samples. In order to make sure that the results are free of finite size effect, we have also performed a simulation on $L=256$ for $r=0.25$ at K_c.

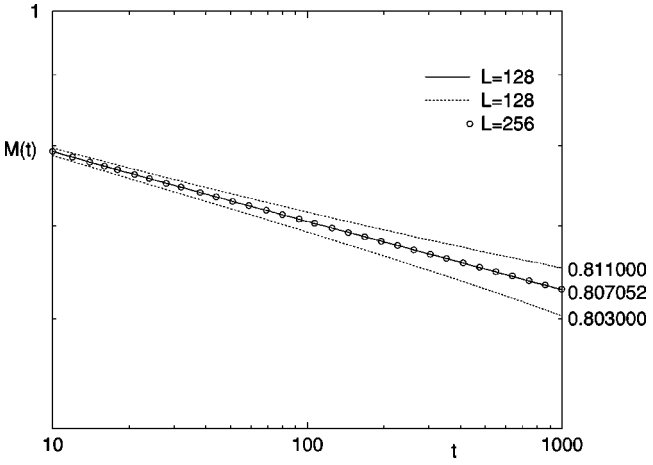

FIG. 1. The time-dependent magnetization starting from ordered state at three different K's in the critical region for $r=0.25$ plotted in double-log scale.

In Fig. 1, the time-dependent magnetization at K_c and $K_\pm=(1\pm0.005)K_c$ for $r=0.25$ is plotted on a double log scale. At the critical point $K_c=0.807\,052$, the magnetization shows a nice power law behavior after a microscopic time scale $t_{mic}\sim100$. The perfect overlap of the two curves for $L=128$ and $L=256$ shows that there is already no finite size effect for $L=128$. From the slope of $M(t)$ at K_c we estimate the exponent $\eta/2z=0.0492(3)$ for $L=128$ and $0.0489(6)$ for $L=256$. Within the errors, they are consistent. For $r=0.5$ and $r=0.1$, the exponent $\eta/2z$ is $0.0554(3)$ and $0.0397(3)$, respectively.

In Fig. 2, the time evolution of $U(t,L)$ for $r=0.25$ with $L=128$ and $L=256$ is plotted on a log-log scale. Both curves show similar power law behavior after $t_{mic}\sim100$. The slopes of the curves give the exponent $d/z=0.789(5)$ for L

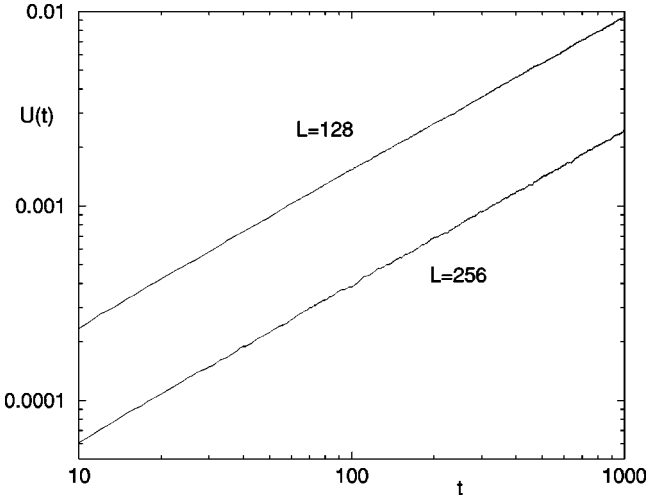

FIG. 2. Binder cumulant for $L=128$ and $L=256$ plotted in log-log scale for $r=0.25$.

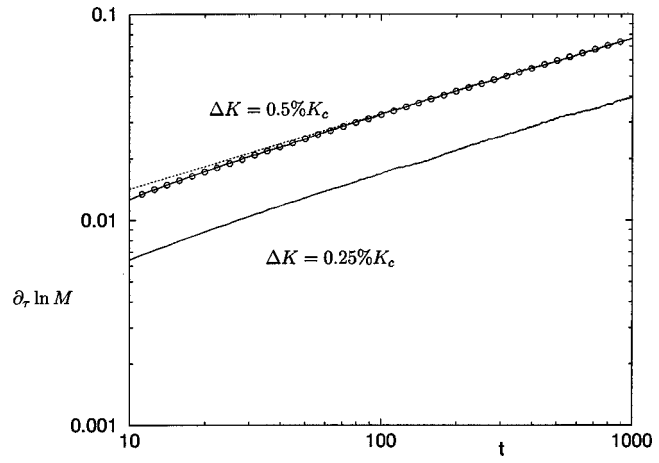

FIG. 3. The derivative $\partial_\tau \ln M(t,\tau)|_{\tau=0}$ at K_c for $r=0.25$. The top curve is estimated from $\Delta K=0.5\%$ of K_c and the bottom from $\Delta K=0.25\%$ of K_c. The power law fit within the time interval [100,1000] is shown as the extended dotted line, while a fit to Eq. (17) on [10,1000] is denoted by circles.

$=128$ and 0.791(8) for $L=256$. The agreement of these two values confirms again that the finite size effect is negligibly small. The exponent d/z for $r=0.5$ and $r=0.1$ are estimated to be 0.879(9), and 0.644(7), respectively. From these results we obtain the dynamic exponent $z=2.28(2)$, 2.53(3), and 3.11(3), respectively, for $r=0.5$, 0.25, and 0.1. It is clear that the dynamic exponent z varies with the strength of disorder r. Taking the dynamic exponent z as input we can calculate the critical exponent η from $\eta/2z$ measured from $M(t)$. For $r=0.5$, 0.25, and 0.1, the exponent η is estimated to be 0.252(3), 0.249(3), and 0.249(3), respectively. These values are consistent with the theoretical value 0.25 of the pure Ising model. This universal η is also supported by all recent numerical measurements in equilibrium.

In Fig. 1 we see that the magnetization curves upwards above K_c and downwards below K_c. With these data, the derivative $\partial_\tau \ln M(t)$ can be approximately calculated. In Fig. 3, the time evolution of this derivative for $r=0.25$ calculated with $\Delta K=0.5\%K_c$ and $\Delta K=0.25\%K_c$ is plotted on a double-log scale (solid lines). Both curves show similar power law behavior. In a time interval [100,1000], we measure from the slopes the critical exponent $1/\nu z=0.368(2)$ for $\Delta K=0.5\%K_c$ and 0.366(5) for $\Delta K=0.25\%K_c$. The agreement of these two values indicates that $\Delta K=0.5\%K_c$ is small enough to estimate the derivative $\partial_\tau \ln M(t)$. For the other two values of the strength of disorder $r=0.5$ and $r=0.1$ we measure $1/\nu z=0.422(3)$ and 0.284(3), respectively.

With the exponent z and $1/\nu z$ in hand, we can calculate the exponent ν. For $r=0.5$, 0.25, and 0.1, ν is 1.04(2), 1.07(2), and 1.13(2), respectively. These values of ν are slightly bigger than 1, and show a small dependence on the strength of randomness. This is qualitatively consistent with the results of Ref. [16], where even bigger values of ν are reported. In Table I, all the exponents are listed in comparison with those for $r=1$ [24]. For $r=1$, $\nu=0.97(2)$ shows that there is still a small systematic error or a correction to scaling in this measurement. Indeed, how to obtain very accurate ν from short-time dynamics, e.g., to the accuracy of 1%, remains open. But this systematic error makes ν smaller and would not explain bigger ν for smaller r. We will come back to this point at the end of this section.

Here we would like to make some more comments. To estimate the exponent ν, we have used the measured exponents z and $1/\nu z$ as input. We are relatively confident of the measurement of z since the critical point is exactly known and the scaling behavior of $U(t,L)$ is not affected by the logarithmic correction in Eq. (5). The reasonable results for the exponent η confirm this. However, the measurement of $1/\nu z$ from the slope of $\partial_\tau \ln M(t)$ could theoretically be affected by the logarithmic correction as shown in Eq. (1).

The power law behavior $\partial_\tau \ln M(t,\tau)|_{\tau=0} \sim t^{1/\nu z}$ is deduced from the scaling form $M(t,\tau)=b^{-\beta/\nu}M(b^{-z}t, b^{1/\nu}\tau)$. If there is a logarithmic correction for $\xi(\tau)$, this scaling form will be modified. Let us start from the following more general scaling form:

$$M(t,\tau)=b^{-\beta/\nu}M(b^{-z}t,\tau'(\tau,b)), \tag{13}$$

where $\tau'(\tau,b)$ should be determined by the relation $\xi(\tau')=b^{-1}\xi(\tau)$. According to Eq. (1), we have

$$\tau'^{-\nu}[1+C\ln(1/\tau')]^{\tilde{\nu}}=b^{-1}\tau^{-\nu}[1+C\ln(1/\tau)]^{\tilde{\nu}}. \tag{14}$$

By writing τ' as $\tau'=\tau b^{1/\nu}f(\tau,b)$, we get

$$f=\left[1-\frac{C\ln(b^{1/\nu}f)}{1+C\ln(1/\tau)}\right]^{\tilde{\nu}/\nu} \approx \left[1-\frac{C\ln(b^{1/\nu})}{1+C\ln(1/\tau)}\right]^{\tilde{\nu}/\nu}, \tag{15}$$

TABLE I. Results for the exponents from simulations starting from both random initial states and an ordered initial state. The exponents ν and ν_c are obtained without and with the taking into account of the corrections to scaling in Eq. (17), respectively. For the calculation of $z=(2-\eta)/y$, the theoretical value $\eta=0.25$ is used as input. Data for $r=1$ are taken from Ref. [24].

	Random start					Ordered start			
r	θ	y	λ	$z=(2-\eta)/y$	$z=d/(\lambda+\theta)$	z	η	ν	ν_c
1	0.191(1)	0.817(7)	0.737(1)	2.14(2)	2.155(3)	2.16(2)	0.249(2)	0.97(2)	0.99(3)
0.5	0.183(2)	0.76(1)	0.69(1)	2.30(3)	2.29(4)	2.28(2)	0.252(3)	1.04(2)	1.06(3)
0.25	0.161(2)	0.69(1)	0.62(1)	2.54(4)	2.56(4)	2.53(3)	0.249(3)	1.07(2)	1.09(4)
0.1	0.127(1)	0.56(1)	0.52(1)	3.12(5)	3.09(5)	3.11(3)	0.249(3)	1.13(2)	1.15(3)

where the approximation is based on a very small τ. From above equations, we obtain

$$\partial_\tau \ln M(t,\tau) \sim t^{1/\nu z}\left[1-\frac{C\ln(t^{1/\nu z})}{1+C\ln(1/\tau)}\right]^{\tilde{\nu}/\nu}. \qquad (16)$$

For a finite τ, there is a logarithmic correction that disappears at the exact critical temperature $\tau=0$.

However, Eq. (16) does not yield a good fit to the data in Fig. 3. Actually, according to Eq. (16) the correction to the power law becomes prominent for large t but the curve in Fig. 3 shows deviation from power law mainly in shorter times. The deviation from a power law mainly in shorter times is usually considered to be the effect of microscopic detail. It can be also described as a kind of correction to scaling. To investigate this correction to scaling, we have fitted our results in the time interval [10,1000] with the following ansatz:

$$\partial_\tau \ln M(t) = a t^{1/\nu z}(1-bt^{-\alpha}). \qquad (17)$$

From Fig. 3 we see that the fit is rather good. The best-fitting values of α are 0.15, 0.20, and 0.22, respectively, for different strengths of randomness $r=0.5$, 0.25, and 0.1. The exponent $1/\nu z$ is reduced about 2% and with a bigger error due to this four-parameter fitting. This means that the correction to the scaling could only increase our values of the exponent ν. Actually, we *are* happy with this 2% correction since then the exponent ν for $r=1$ comes very close to 1, and it explains the systematic error for ν mentioned before, even though statistical errors become bigger. In Table I, the values for the exponent ν after taking into account the corrections to scaling are given as ν_c. With these discussions we should state that our results favor the weak universality scenario with an r-dependent exponent ν.

IV. QUENCHED FROM A HIGH TEMPERATURE STATE

For the dynamic process starting from a high temperature state, our numerical data show that there are rather strong dynamic corrections to scaling, as in the 2D XY model [30,31]. It is interesting that if we perform the measurements in a time interval, e.g., [100,2000], all data result consistently in a roughly correct η, but the dynamic exponent z is around 5–10 % bigger than that obtained from a dynamic process starting from an ordered initial state discussed in the last section. The corrections to scaling seem not to change the static exponents and the scaling relations between the exponents. It is very important to clarify these corrections to scaling and how they look like.

We start from the measurements of the exponent θ. Here we will *not* consider the corrections to scaling. It is not that we are sure there are no corrections to scaling but it is too difficult to perform simulations to longer times. We need very small initial magnetization m_0 and large lattices. Fortunately, the value of the exponent θ is relatively small and the correction of it will not affect so much other exponents in our data analysis. As indicated in Eq. (5), starting from a high temperature state with a small initial magnetization m_0, the magnetization of the system undergoes an initial power law increase, described by the exponent θ. We have measured this exponent for different strengths of randomness $r=0.5$, 0.25, and 0.1. In the simulations, initial configurations are prepared simply by setting the spin to be 1 or -1 on each lattice site randomly with the probability $(1+m_0)/2$ and $(1-m_0)/2$, respectively. After generating an initial configuration, the system is then updated with the Metropolis algorithm up to 200 Monte Carlo steps. The lattice size is $L=128$ and the initial magnetization is $m_0=0.02$. For each r, the average is over 300 realizations of the distribution of couplings, and for each realization the simulation is carried out for 500 independent initial configurations.

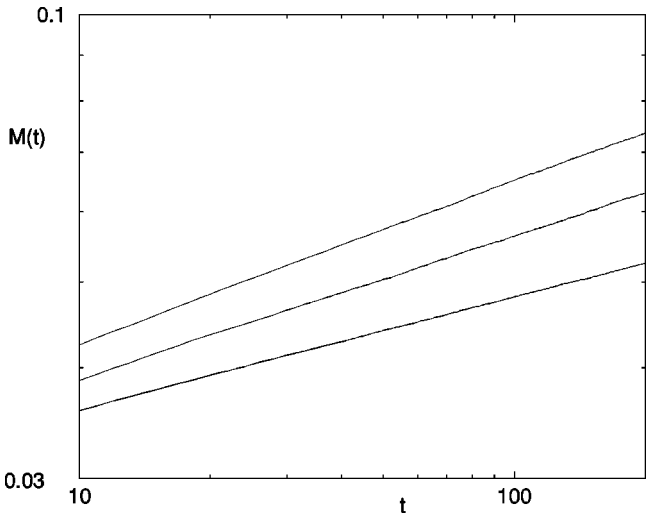

FIG. 4. Time evolution of the magnetization for different strengths of randomness r at their critical temperatures plotted in double-log scale. From top to bottom, the r is 0.5, 0.25, and 0.1, respectively.

In Fig. 4 the time evolutions of the magnetization for different strengths of randomness r at their critical temperatures are plotted on double-log scale. The magnetization indeed increases. If we measure the slopes of the curves in a time interval [10,200] the critical exponent θ for $r=0.5$, 0.25, and 0.1 are 0.183(2), 0.161(2), and 0.127(1), respectively. The results are smaller than that of the 2D Ising model $\theta=0.191(3)$ [36] and are apparently r dependent.

The exponent θ is actually defined in the limit $m_0\to 0$. In order to make sure that in our simulations the initial magnetization m_0 is small enough, we have performed a simulation with $m_0=0.01$ for $r=0.25$. The measured value of θ is 0.162(4) and in good agreement with $\theta=0.161(2)$ with $m_0=0.02$. In our measurements, the errors induced by the finite initial magnetization m_0 can be ignored.

Now let us turn to dynamic evolution from a random initial state with $m_0=0$. In this dynamic process, the autocorrelation $A(t)$ and the second moment $M^{(2)}(t)$ are expected to have power law behavior as described in Eqs. (8) and (6).

Some preliminary results show that in this evolution, the dynamic scaling behavior of the autocorrelation $A(t)$ and the second moment $M^{(2)}(t)$ suffer from severe corrections to scaling, which increase for smaller r. In order to investigate clearly the corrections to scaling, we perform these simula-

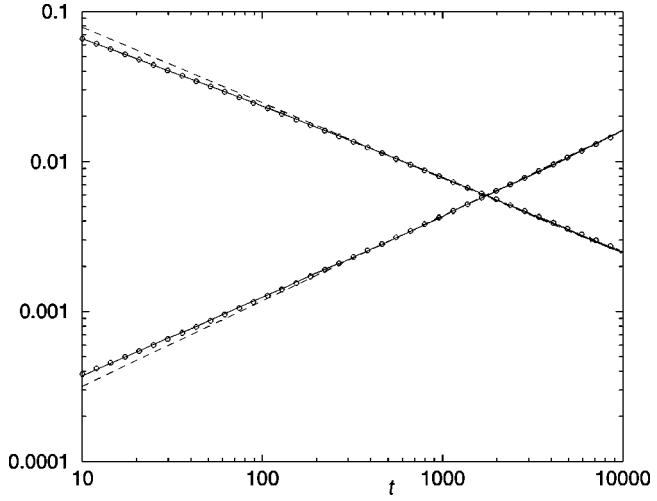

FIG. 5. Time evolution of the autocorrelation $A(t)$ and the second moment $M^{(2)}$ for $r=0.1$ at K_c plotted in double-log scale. The extended dashed lines show the power law fits within the time interval [500,10 000], while the fits to Eqs. (18) and (19) on [10,10 000] are denoted by circles.

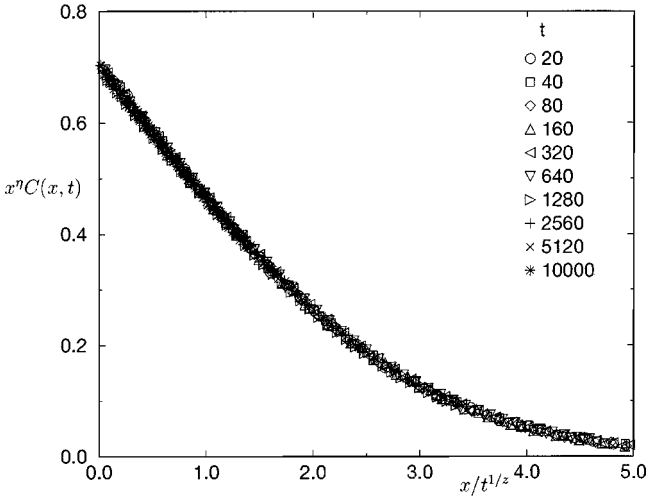

FIG. 6. The scaling function $g(x/t^{1/z})=x^\eta C(x,t)$ for $r=0.25$.

tions up to 10 000 Monte Carlo steps. To avoid the finite size effect the lattice sizes are set to be $L=512$, 512, and 256, respectively, for the three different strengths of randomness $r=0.5$, 0.25, and 0.1. Such different choices are based on the fact that the dynamic exponent z increases for smaller r, and for $r=0.1$ there is already no finite size effect with $L=256$ until 10 000 Monte Carlo steps. The averages are over 500 realizations of the distribution of couplings. For each realization, the simulation is carried out with 10 and 50 independent initial configurations for $L=512$ and $L=256$, respectively.

In Fig. 5, the time evolution of the autocorrelation $A(t)$ and the second moment $M^{(2)}(t)$ for $r=0.1$ at K_c are plotted as solid lines on double-log scale. Since the corrections to scaling are severe, we perform a power law fit in a time interval [500,10 000]. The results are shown with dashed lines. With these power law fits, the exponents $y=(2-\eta)/z$ and λ are measured to be 0.566(3) and 0.503(3), respectively for $r=0.1$. To investigate the effect of the correction to scaling systematically, we fit the autocorrelation $A(t)$ in the time interval [10,10 000] with the following ansatz:

$$A(t)=at^{-\lambda}(1-bt^{-c}), \tag{18}$$

and similarly fit the second moment $M^{(2)}(t)$ with

$$M^{(2)}(t)=at^{y}(1-bt^{-c}). \tag{19}$$

The best-fitting values of c are about 0.2 and 0.4, respectively, for $A(t)$ and $M^{(2)}(t)$. The resulting exponents $y=(2-\eta)/z$ and λ are 0.56(1) and 0.52(1). Compared with those obtained from the power law fits starting from $t_{mic}=500$, a different value for λ shows that the correction to scaling is strong and $t_{mic}=500$ cannot remove all its effect. We have also tried the ansatz with logarithmic corrections proposed for the 2D XY model in Ref. [31]. The fits seem also not bad, and the quality is similar to that for the fits with Eqs. (18) and (19), but the resulting exponents are not reasonable. For $r=0.1$, for example, we obtain $y=0.62(1)$ from the second moment and $\lambda=0.55(1)$ from the autocorrelation. These values are too high compared to $y=0.56(1)$ and $\lambda=0.52(1)$ from the fits with Eqs. (18) and (19).

With the exponent $y=(2-\eta)/z$ in hand, we can estimate the dynamic exponent z by using the theoretical value $\eta=0.25$ as an input. The latter has been verified by the simulations from an ordered initial state. On the other hand, we can also calculate the dynamic exponent z from the relation $z=d/(\lambda+\theta)$ by using the exponents θ and λ as input. For $r=0.1$, with λ and y measured by fitting to Eqs. (18) and (19), we get $z=3.09(5)$ and 3.12(5). The two values agree well with each other and are consistent with $z=3.11(3)$ measured in the simulation starting from an ordered initial state. Here we should mention that the correction of θ will not change the estimate of the dynamic exponent z so much, since θ is much smaller than λ.

For the other two strengths of randomness $r=0.5$ and 0.25, the exponent $y=(2-\eta)/z$ are measured to be 0.76(1) and 0.69(1), respectively, by fitting to Eq. (19), while the exponent λ is 0.69(1) and 0.62(1) by fitting to Eq. (18).

All the exponents calculated from our measurements are listed in Table I. The exponents of the pure Ising model ($r=1$) measured with short-time dynamic simulations are also given in Table I for comparison (see the review paper [24] and references therein). We can see that the dynamic exponents calculated from the slope of the second moment $z=(2-\eta)/y$ and from $z=d/(\lambda+\theta)$ are in good agreement with the value measured from ordered start for all different strengths of randomness r.

To confirm the short-time dynamic scaling, we have also measured the equal-time correlation function $C(x,t)$ in these simulations. The $C(x,t)$ should be subject to the following dynamic scaling:

$$C(x,t)=x^{-\eta}g(x/t^{1/z}). \tag{20}$$

Based on this equation we can extract the exponents z and η independently from the correlation function $C(x,t)$ by searching for the best scaling collapse.

As discussed above, the correction to scaling gets stronger for smaller r. Therefore, in performing the scaling collapse, we must set proper t_{mic}. In case of the second moment, t_{mic} should be about 500, 200, and 100 for $r=0.1$, 0.25, and 0.5, respectively. For the scaling collapse of $C(x,t)$, t_{mic} can be slightly smaller. For example, in Fig. 6 we present the scaling function $g(x/t^{1/z})=x^{\eta}C(x,t)$ for $r=0.25$. By searching for the best scaling collapse in a time interval [80,10 000], we get $\eta=0.250(1)$ and $z=2.53(2)$. These results are very reasonable. For $r=0.5$, we get $\eta=0.250(1)$ and $z=2.28(1)$ with $t_{mic}=40$, and for $r=0.1$ we get $\eta=0.250(2)$ and $z=3.11(1)$ with $t_{mic}=320$. All these results are in very good agreement with the former measurements.

V. CONCLUSIONS

We have systematically investigated the short-time critical dynamic behavior of the two-dimensional random-bond Ising model with Monte Carlo methods. The simulations are carried out for different strengths of randomness and for different initial conditions. Scaling behavior has been examined and corrections to scaling are seriously considered. The exponents θ and z are found to be the strength of randomness dependent. The exponent η is very consistent with the Ising value $\eta=0.25$, but the exponent ν is slightly bigger than 1 and is dependent on the strength of randomness.

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